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Multiple Scattering among Vias in Planar Waveguides Using SMCG Method

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Abstract— **Large scale full-wave solution of multiple scattering among cylindrical vias in planar waveguides is modeled using Foldy-Lax equations. Solution of the Foldy-Lax equations with large number of unknowns is done efficiently using the sparse-matrix canonical-grid method. In the method, interactions among vias are decomposed into strong interactions part and weak interactions part where the calculation can be carried out using 2D-FFT after the locations of the vias have been translated onto the uniform grids. The final solution of the Foldy-Lax equations is calculated by iterative method with matrix-vector multiplication speeded up by the 2D-FFT operation. The results show $O(N \log N)$ CPU efficiency and $O(N)$ memory efficiency and make large scale via problem possible for computer simulation.**

I. Introduction

In recent years, there has been increasing interests on the electromagnetic effects of using vias on the high speed interconnect design. For sub-GHz frequency PCB interconnect analysis, effects of single vias are usually approximated by equivalent RLC circuit model. For higher frequency, the concern is that quasi-static or equivalent circuit model would not be accurate enough for practical application, and it becomes necessary to resort to full wave analysis for more accurate modeling of the same problem. From the electromagnetic perspective, as signal vias are excited, parallel plate waveguide effects are induced by the multi-layered geometry; signals on active vias can excite waveguide modes within layers and hence can affect other separated vias. As a second order effect, the affected vias can in turn influence the original signal. Because of the waveguide modes, such coupling is not necessarily localized in space. This poses considerable design problems for reliability, high speed, and simulation. Such coupling can even cause unreliable behavior or complete signal failure, along with signal integrity loss, higher delays, and inappropriate switching of signals. In published results, methods like FDTD [1] and moment methods [2][3] have been used for the via modeling. While those methods offer good accuracy for a few number of vias, they are computationally prohibitive for large scale problems. Simpler models using quasi-static electromagnetic analysis [4]-[8] have also been developed. Recently, we used a semi-analytical technique of Foldy-Lax[9] equations to compute the full wave solution of multiple scattering cylindrical vias in planar waveguides. The waveguide modes are decoupled in the Foldy- Lax equations so that the solution can be calculated for each waveguide mode separately. It has been shown in [9] that the approach agrees with the results from other literatures. In practical applications, tens of thousands of vias are commonly seen on a printed circuit board. Solving the Foldy-Lax equation using direct matrix inversion would quickly deplete the computer resource because its CPU dependency is of $O(N^3)$ and memory requirements of $O(N^2)$, making large scale via problem intractable. In dealing with large scale electromagnetic problems, researchers have developed new algorithms that will dramatically reduce the complexity of many electromagnetic from polynomial order to logarithmic[12][13][14] order or even linear order[16][15]. In this paper, a sparse-matrix canonical-grid method(SMCG)[12] is used to solve the Foldy-Lax multiple scattering equation efficiently. The method decomposes the all interactions into strong interaction(near field) part and weak interaction(far field) part. For strong interaction part, field solution are calculated directly based on the Foldy-Lax equations. For the weak interaction part, which accounts for most of the CPU time, the field solution is calculated indirectly by expanding the original field about uniform grids so that FFT can be used to speed up the computation. As a result, we can solve Foldy-Lax multiple scattering equations with $O(N \log N)$ CPU efficiency and $O(N)$ memory efficiency.

II. Foldy- Lax equations for scattering with Multiple Cylinders

Consider N vias placed between two parallel plates centered at $\overline{\rho}_1, \overline{\rho}_2 .... \overline{\rho}_N$ as shown in Figure 1. The solution is expressed in terms of waveguide modes, and also in terms of vector cylindrical waves for modal representation of field in the region between the plates. The multiple scattering can be formulated in terms of Fold-Lax multiple scattering equations. The details can found in our pervious work in [9][10]. The Foldy- Lax equations state that the final exciting field of cylinder q is equal to the incident field plus scattered fields from all cylinders except the scattered field from itself. The scattered field will be incident p to cylinder q can be re-expressed by using translation addition theorem. Thus the Foldy-Lax multiple scattering equations for TM polarization are in the following form

$$w_{\ell n}^{TM(q)} = a_{\ell n}^{TM(q)} + \sum_{\substack{p=1 \\ p \neq q}}^{N} \sum_{m=-\infty}^{\infty} H_{n-m}^{(2)}(k_{\rho \ell} |\overline{\rho}_p - \overline{\rho}_q|) e^{j(n-m)\phi_{\overline{\rho}_p \overline{\rho}_q}} \cdot T_m^{(N)} w_{\ell m}^{TM(p)} \qquad (1)$$

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where $a_{\ell n}^{TM(q)}$ is the incident field of the current source onto cylinder q and is given in. In the Foldy- Lax equations there is no coupling between different ℓ because each ℓ corresponds to a specific k_z. Neither is there coupling between TE and TM waves because the cylinders are perfectly conducting.

For the general case with N cylinders with voltages $V_{1u}, V_{2u}, \ldots\ldots V_{Nu}, V_{1b}, V_{2b}, \ldots\ldots$ and V_{Nb}. Then, we have, for sources at $z' = d/2$, that is $V_{1u}, V_{2u}, \ldots\ldots V_{Nu}$,

$$
\begin{aligned}
& a_{\ell n}^{TM(q)} \qquad (2) \\
= & \frac{jk}{2d}\frac{(-1)^{n+\ell}}{k_{\rho\ell}^2} f_\ell \frac{2\pi V_{qu}}{\ln\frac{b}{a}}\delta_{n0}\left[H_0^{(2)}(k_{\rho\ell}b) - H_0^{(2)}(k_{\rho\ell}a)\right] \\
& + \sum_{j\neq q}^{N} \frac{jk}{2d}\frac{(-1)^{n+\ell}}{k_{\rho\ell}^2} f_\ell H_n^{(2)}\left(k_{\rho\ell}\left|\overline{\rho}_q - \overline{\rho}_j\right|\right) e^{jn\phi_{\overline{\rho}_q\overline{\rho}_j}} \\
& \left[\frac{2\pi V_{ju}}{\ln\frac{b}{a}}\left[J_0(k_{\rho\ell}b) - J_0(k_{\rho\ell}a)\right]\right]
\end{aligned}
$$

By solving Foldy- Lax equations for $w_{\ell n}^{TM(q)}$, we can find currents at the cylinders at $z = +d/2$. That will be current I^{uu},that is current at $z = d/2$ due to source at $z' = d/2$

$$
I^{(p)uu} = 2\pi a \sum_{\ell} w_{\ell 0}^{TM(p)} k_{\rho\ell} \frac{\frac{2}{\pi k_{\rho\ell} a}}{H_0^{(2)}(k_{\rho\ell}a)} \frac{1}{\eta}(-1)^\ell \qquad (3)
$$

Then we find currents at the cylinders at $z = -d/2$. That will be current I^{bu}, *that* is current at $z = -d/2$ due to source at $z\prime = d/2$

$$
I^{(p)bu} = 2\pi a \sum_{\ell} w_{\ell 0}^{TM(p)} k_{\rho\ell} \frac{\frac{2}{\pi k_{\rho\ell} a}}{H_0^{(2)}(k_{\rho\ell}a)} \frac{1}{\eta} \qquad (4)
$$

III. Applying Sparse-Matrix Canonical-Grid Method(SMCG) to Solve the Foldy-Lax Equation

In the SMCG method, the field is decomposed into strong interaction part and weak interaction part as shown in Figure 2. The strong (near field) interactions, which is defined for the neighboring vias within a radius, are calculated directly from the Foldy-Lax equations. For the weak(far field) interactions, which account for most computation time, the computation is done indirectly via translation to the canonical grid points by using the translational addition theorem. Figure 2 illustrates the process of indirect calculation for the weak interaction between via p and via q. First, the local-to-grid expansion($p \rightarrow p_0$) translates the scattered field from p to its nearest grid p_0 by using the translational addition theorem, then followed a grid-to-grid($p_0 \rightarrow q_0$) operation which is carried out on the canonical grids. Lastly, the scattered field originating from p is now collected in q_0 and to be distributed to the observing via q by a grid-to-local expansion($q_0 \rightarrow q$), where the translation addition theorem is again used. Since the Green's function in our problem is translational invariant, one can observe that the grid-to-grid operations on the uniform grids can be evaluated efficiently using 2D-FFT in $O(N_g \log N_g)$ CPU efficiency and $O(N)$ memory efficiency, where N_g is the number of canonical grids and N is the total number of vias. Note that it is possible for the same grid point to associate with more than one via. In this case, the field coming from different vias are collected (added) together. For SMCG algorithm, we rewrite the Foldy-Lax equations (1):

$$
\begin{aligned}
w_{\ell n}^{TM(q)} = & \; a_{\ell n}^{TM(q)} \qquad (5) \\
& + \sum_{\substack{p\notin \mathbf{S(p)} \\ p\neq q}}^{N} \sum_{m=-\infty}^{\infty} \sum_{m'=-\infty}^{\infty} \sum_{n'=-\infty}^{\infty} T_m w_{\ell m}^{TM(p)} \cdot \\
& J_{m'-m}(k_{\rho\ell}\left|\overline{\rho}_p - \overline{\rho}_{p_0}\right|) e^{j(m'-m)\phi_{\overline{\rho}_p\overline{\rho}_{p_0}}} \cdot \\
& H_{n'-m'}^{(2)}(k_{\rho\ell}\left|\overline{\rho}_{p_0} - \overline{\rho}_{q_0}\right|) e^{j(n'-m')\phi_{\overline{\rho}_{p_0}\overline{\rho}_{q_0}}} \cdot \\
& J_{n-n'}(k_{\rho\ell}\left|\overline{\rho}_{q_0} - \overline{\rho}_q\right|) e^{j(n-n')\phi_{\overline{\rho}_{q_0}\overline{\rho}_q}} \\
& + \sum_{\substack{p\in \mathbf{S(p)} \\ p\neq q}} \sum_{m=-\infty}^{\infty} H_{n-m}^{(2)}(k_{\rho\ell}\left|\overline{\rho}_p - \overline{\rho}_q\right|) e^{j(n-m)\phi_{\overline{\rho}_p\overline{\rho}_q}} \cdot \\
& T_m^{(N)} w_{\ell m}^{TM(p)}
\end{aligned}
$$

where $\mathbf{S(p)}$ represents the set of vias in the neighborhood of via p. Moreover, a via q is said to be in the neighborhood of via p if it satisfies

$$
\left|\overline{\rho}_{p_0} - \overline{\rho}_{q_0}\right| \leq n_d \qquad (6)
$$

For one combination of (m, n, m', n') indices, (5) can be rewritten in matrix form as

$$
\overline{\overline{Z}}\,\overline{w} = \overline{a} \qquad (7)
$$

$$
\overline{\overline{Z}} = \overline{\overline{Z}}^{(S)} + \overline{\overline{Z}}^{(W)} \qquad (8)
$$

$$
\overline{\overline{Z}}^{(W)} = \overline{\overline{L}}^{(d)}\overline{\overline{Z}}^{(G)}\overline{\overline{L}}^{(u)} \qquad (9)
$$

where $\overline{\overline{Z}}^{(S)}$ and $\overline{\overline{Z}}^{(W)}$ correspond to strong and weak interaction, $\overline{w}$ and $\overline{a}$ are the vector form of $w_{\ell n}^{(q)}$ and $a_{\ell n}^{(q)}$.And,

$$
L_{p_0,q}^{(u)} = \begin{cases} 0; \text{ for } p_0 \neq q_0 \\ T_m J_{m'-m}(k_{\rho\ell}\left|\overline{\rho}_q - \overline{\rho}_{q_0}\right|) \cdot \\ e^{j(m'-m)\phi_{\overline{\rho}_q\overline{\rho}_{q_0}}}; \text{ for } p_0 = q_0 \end{cases}
$$

$$
Z_{p_0,q_0}^{(G)} = \begin{cases} 0; \text{ for } \left|\overline{\rho}_{p_0} - \overline{\rho}_{q_0}\right| \leq n_d \\ -H_{n'-m'}^{(2)}(k_{\rho\ell}\left|\overline{\rho}_{p_0} - \overline{\rho}_{q_0}\right|) \cdot \\ e^{j(n'-m')\phi_{\overline{\rho}_{p_0}\overline{\rho}_{q_0}}}; \text{ otherwise} \end{cases}
$$

$$
L_{p,q_0}^{(d)} = \begin{cases} 0; \text{ for } p \neq q \\ J_{n-n'}(k_{\rho\ell}\left|\overline{\rho}_{q_0} - \overline{\rho}_q\right|) \\ e^{j(n-n')\phi_{\overline{\rho}_{q_0}\overline{\rho}_q}}; \text{ for } p = q \end{cases}
$$

$$Z_{p,q}^{(S)} = \begin{cases} -H_{n-m}^{(2)}(k_{\rho\ell}\left|\overline{\rho}_p - \overline{\rho}_q\right|)e^{j(n-m)\phi_{\overline{\rho_p\rho_q}}} \cdot \\ T_m; \text{ for } \left|\overline{\rho}_{p_0} - \overline{\rho}_{q_0}\right| \leq n_d \text{ and } p \neq q \\ 1; \; p = q \text{ and } m = n \end{cases}$$

It can be seen that $\overline{\overline{L}}^{(u)}$ is simply a local-to-grid operation of $O(N)$ CPU/memory efficiency since it maps N local vias to their corresponding grids(i.e. $\overline{\overline{L}}^{(u)}$ only has N nonzero elements), and $\overline{\overline{L}}^{(d)}$ is a grid-to-local operation of $O(N)$ CPU/memory efficiency doing the reverse mapping from the grids to the vias. Also recognized is that $\overline{\overline{Z}}^{(G)}$ is a block toeplitz matrix when casted in two dimension grid indices. This means the storage of $\overline{\overline{Z}}^{(G)}$ only requires $O(N_g)$, and since the kernel function in $Z_{p0,q0}^{(G)}$ is translational invariant, a $\overline{\overline{Z}}^{(G)}\overline{X}$ multiplication can be facilitated by using 2D-FFT with $O(N_g \log N_g)$ CPU efficiency.

IV. Matrix Notation for Admittance Matrix $\overline{\overline{Y}}$ of Interior Problem

After Foldy-Lax equation is solved using SMCG method, the currents can be computed. If we define current vectors of dimension N, $\overline{I}^{uu}$, which is current at upper aperture of $z = d/2$ due to sources at $z' = d/2$.Also, current vectors of dimension N, $\overline{I}^{bu}$, which is current at lower aperture of $z = -d/2$ due to sources at $z' = d/2$.

$$\begin{aligned} \overline{I}^{uu} &= \overline{\overline{Y}}^{uu}\overline{V}^{u} \\ \overline{I}^{bu} &= \overline{\overline{Y}}^{bu}\overline{V}^{u} \end{aligned}$$

Each column of the admittance matrix can be solved individually under a particular port excitation condition.

$$\overline{\overline{Y}}_{\cdot j}^{uu} = \overline{I}^{uu}\Big|_{V_j^u=1, V_k^u=0 \text{ with } k\neq j, V_i^b=0 \text{ for all } i} \tag{10}$$

$$\overline{\overline{Y}}_{\cdot j}^{bu} = \overline{I}^{bu}\Big|_{V_j^u=1, V_k^u=0 \text{ with } k\neq j, V_i^b=0 \text{ for all } i} \tag{11}$$

where $\overline{\overline{Y}}^{uu}$ and $\overline{\overline{Y}}^{bu}$ are of dimension NxN, $\overline{\overline{Y}}_{\cdot j}^{uu}$ and $\overline{\overline{Y}}_{\cdot j}^{bu}$ are the j-th column of $\overline{\overline{Y}}^{uu}$ and $\overline{\overline{Y}}^{bu}$, respectively. Also, $\overline{I}^{uu}$ and $\overline{I}^{bu}$ can be computed based on (3), (4) and the exciting coefficients $\overline{w}_\ell$ solved by SMCG method.

It has been shown in[9] that the admittance matrix can be expressed as

$$\overline{\overline{Y}} = \begin{bmatrix} \overline{\overline{Y}}^{uu} & -\overline{\overline{Y}}^{ub} \\ -\overline{\overline{Y}}^{ub} & \overline{\overline{Y}}^{uu} \end{bmatrix} \tag{12}$$

The conversion from the admittance matrix to the scattering matrix can be done by

$$\overline{\overline{S}} = \left(Y_0\overline{\overline{I}} + \overline{\overline{Y}}\right)^{-1}\left(Y_0\overline{\overline{I}} - \overline{\overline{Y}}\right) \tag{13}$$

where Y_0 is the characteristic port admittance and $\overline{\overline{I}}$ is the identity matrix of $2N$ by $2N$.

V. Numerical Results and Discussion

In implementing the SMCG algorithm, the number of canonical grids N_g is maintained roughly proportional to the number of vias N so that the grids are refined when the number of vias increases. As a result, each iteration will have $O(N \log N)$ CPU efficiency. In Table 1 and 2, the solution of the Foldy-Lax equations for the exciting coefficients are carried out using SMCG method and direct matrix inversion, respectively. The tolerance of the residue norm is set to 10^{-4} for conjugate gradient method in SMCG solution. For the SMCG method(Table 1), the total CPU time roughly scales as $O(N^2 \log N)$ as the per iteration time $O(N \log N)$ is multiplied by the number of iteration showing an $O(N)$ dependence. The memory efficiency scales in $O(N)$ as expected. For matrix inversion method(Table 2), the total CPU time consists of matrix filling time with roughly $O(N^2)$ dependence and matrix inversion time of $O(N^3)$ dependence. For large number of vias(>2500), it is obvious that the CPU efficiency is dominated by $O(N^3)$.The memory of the matrix inversion method scales in $O(N^2)$ as the full matrix size needs to be stored. Note that the last three rows of Table 2 are extrapolated data based on the the dependence aforementioned, the actual simulation requires more memory than what we have on the current machine(Pentium 800Mhz, 256 MB RAM). In all cases, the SMCG outperforms the matrix inversion method in both CPU and memory efficiency. More importantly, the linear memory dependence in SMCG method allows us to solve a much larger scale multiple scattering via problem while the direct matrix inversion method quickly saturates the machine capacity in a few thousand vias for most machine.

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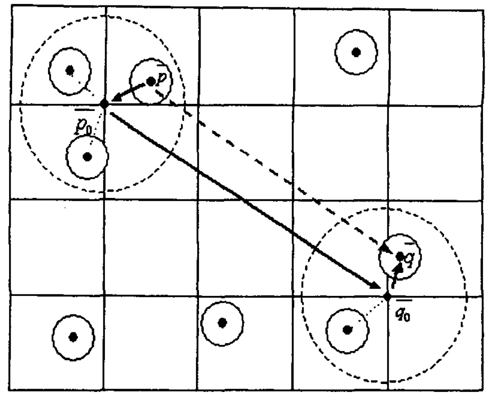

Fig. 2. Direct far field interactions(dashed arrow) between two vias are calculated indirectly through translation to the regular grid points(solid arrows). The near field interactions within a defined radius are computed directly. For the case shown above, the near field radius is , and the interactions within the two dashed circles are calculated directly. Note that it is possible for more than one cylinder to be associated with the same gird.

# of vias	Total CPU	Matrix Filling	Inv Time	Memory
512	144	116	28	8
1024	805	581	224	32
2048	3335	2240	1094	128
4096	22090	13339	8752	512
8192	158379	88363	70016	2048
16384	1193645	633517	560128	8192

Table 2: CPU time(second)/Memory(MB) scaling based on direct matrix inversion.

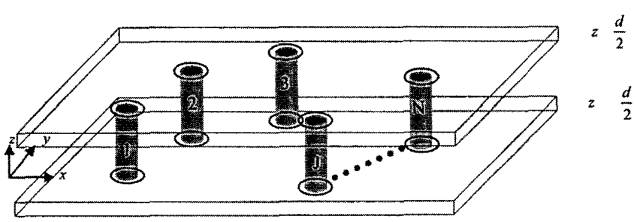

Fig. 1. N randomly placed vais between two parallel metal planes.

# of vias	CPU per iter.	# of iter.	Total CPU	Setup Time	Memory
512	0.2034	59	40	28	0.078
1024	0.4545	121	113	58	0.156
2048	0.9106	246	344	120	0.312
4096	2.0353	482	1231	250	0.624
8192	4.3211	984	4753	501	1.248
16384	9.2143	1988	19329	1011	2.496

Table 1: CPU time(second)/Memory(MB) scaling based on SMCG Method